Oxford Cambridge and RSA

Wednesday 25 May 2022 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes

9458422014

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

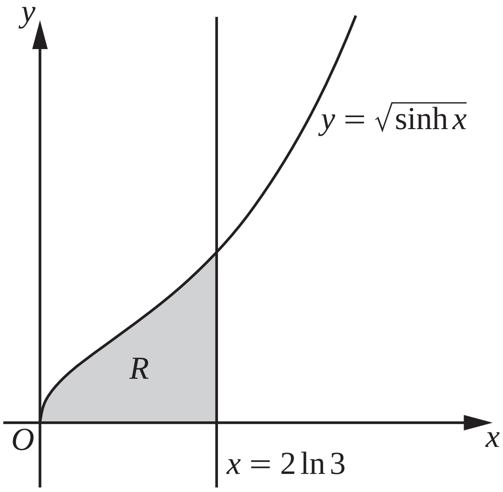
ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

(a) Show that $\cosh(2\ln 3) = \frac{41}{9}$. **[2]**

The region R is bounded by the curve with equation $y = \sqrt{\sinh x}$, the x-axis and the line with equation $x = 2\ln 3$ (see diagram). The units of the axes are centimetres.



A manufacturer produces bell-shaped chocolate pieces. Each piece is modelled as being the shape of the solid formed by rotating R completely about the x-axis.

(b) Determine, according to the model, the exact volume of one chocolate piece. **[4]**

2 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}$.

(a) Calculate det **A**. **[1]**

(b) Write down $\mathbf{A}^{-1}$. **[1]**

(c) Hence solve the equation $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. **[2]**

(d) Write down the matrix **B** such that $\mathbf{AB} = 4\mathbf{I}$. **[1]**

Matrices **C** and **D** are given by $\mathbf{C} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 0 & 2 & p \end{pmatrix}$ where p is a constant.

(e) Find, in terms of p,

- the matrix **CD**
- the matrix **DC**. **[3]**

It is observed that $\mathbf{CD} \neq \mathbf{DC}$.

(f) The result that $\mathbf{CD} \neq \mathbf{DC}$ is a counter example to the claim that matrix multiplication has a particular property. Name this property. **[1]**

3 **In this question you must show detailed reasoning.**

(a) Find the roots of the equation $2z^2 - 2z + 5 = 0$. **[2]**

The loci C_1 and C_2 are given by $|z| = |z - 2\mathrm{i}|$ and $|z - 2| = \sqrt{5}$ respectively.

(b) (i) Sketch on a single Argand diagram the loci C_1 and C_2, showing any intercepts with the imaginary axis. **[3]**

(ii) Indicate, by shading on your Argand diagram, the region

$$\{z : |z| \leqslant |z - 2\mathrm{i}|\} \cap \{z : |z - 2| \leqslant \sqrt{5}\}.$$ **[1]**

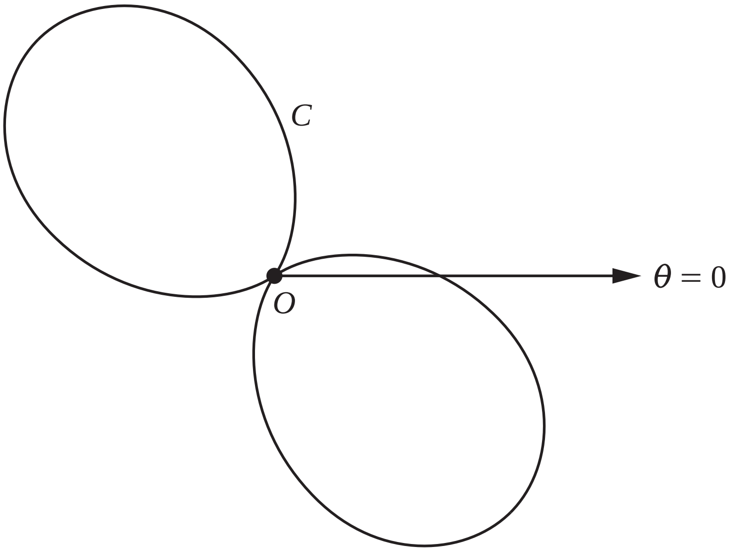
(c) (i) Show that both of the roots of the equation $2z^2 - 2z + 5 = 0$ satisfy $|z - 2| < \sqrt{5}$. **[2]**

(ii) State, with a reason, which root of the equation $2z^2 - 2z + 5 = 0$ satisfies $|z| < |z - 2\mathrm{i}|$. **[1]**

(d) On the same Argand diagram as part **(b)**, indicate the positions of the roots of the equation $2z^2 - 2z + 5 = 0$. **[2]**

4 Determine the acute angle between the line $\mathbf{r} = \begin{pmatrix} -\sqrt{3} \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2\sqrt{3} \\ -\sqrt{3} \end{pmatrix}$ and the y-axis. **[4]**

5 The diagram below shows the curve C with polar equation $r = 3(1-\sin 2\theta)$ for $0 \leqslant \theta \leqslant 2\pi$.



(a) Show that a cartesian equation of C is $(x^2+y^2)^3 = 9(x-y)^4$. **[3]**

(b) Show that the line with equation $y = x$ is a line of symmetry of C. **[2]**

(c) **In this question you must show detailed reasoning.**

Find the exact area of each of the loops of C. **[6]**

6 Let $y = x\cosh x$.

Prove by induction that, for all integers $n \geqslant 1$, $\dfrac{\mathrm{d}^{2n-1}y}{\mathrm{d}x^{2n-1}} = x\sinh x + (2n-1)\cosh x$. **[6]**

7 **(a)** Determine the values of A, B, C and D such that $\dfrac{x^2+18}{x^2(x^2+9)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+9}$. **[4]**

(b) **In this question you must show detailed reasoning.**

Hence determine the exact value of $\displaystyle\int_3^{\infty} \frac{x^2+18}{x^2(x^2+9)}\,\mathrm{d}x$. **[6]**

8 A biologist is studying the effect of pesticides on crops. On a certain farm pesticide is regularly applied to a particular crop which grows in soil. Over time, pesticide is transferred between the crop and the soil at a rate which depends on the amount of pesticide in both the crop and the soil. The amount of pesticide in the crop after t days is x grams. The amount of pesticide in the soil after t days is y grams. Initially, when $t = 0$, there is no pesticide in either the crop or the soil.

At first it is assumed that no pesticide is lost from the system. The biologist further assumes that pesticide is added to the crop at a constant rate of k grams per day, where $k > 6$.

After collecting some initial data, the biologist suggests that for $t \geqslant 0$, this situation can be modelled by the following pair of first order linear differential equations.

$$\frac{dx}{dt} = -2x + 78y + k$$
$$\frac{dy}{dt} = 2x - 78y$$

(a) **(i)** Show that $\frac{d^2x}{dt^2} + 80\frac{dx}{dt} = 78k$. **[2]**

(ii) Determine the particular solution for x in terms of k and t. **[7]**

If more than 250 grams of pesticide is found in the crop, then it will fail food safety standards.

(iii) The crop is tested 50 days after the pesticide is first added to it.

Explain why, according to this model, the crop will fail food safety standards as a result of this test. **[1]**

Further data collection suggests that some pesticide decays in the soil and so is lost from the system. The model is refined in light of this data. The particular solution for x for this refined model is

$$x = k\left(20 - e^{-41t}\left(20\cosh\left(\sqrt{1677}\,t\right) + \frac{819}{\sqrt{1677}}\sinh\left(\sqrt{1677}\,t\right)\right)\right).$$

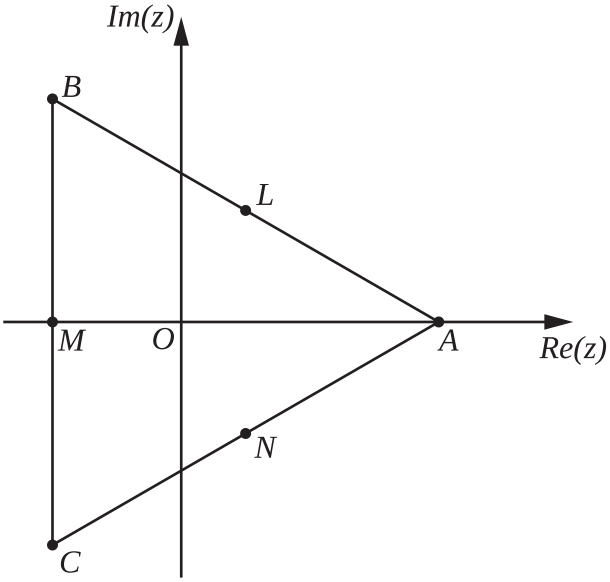
(b) Given now that $k < 12$, determine whether the crop will fail food safety standards in the long run according to this refined model. **[2]**

In the refined model, it is still assumed that pesticide is added to the crop at a constant rate.

(c) Suggest a reason why it might be more realistic to model the addition of pesticide as not being at a constant rate. **[1]**

Turn over for Question 9

9 The cube roots of unity are represented on the Argand diagram below by the points A, B and C.



The points L, M and N are the midpoints of the line segments AB, BC and CA respectively.

Determine a degree 6 polynomial equation with integer coefficients whose roots are the complex numbers represented by the points A, B, C, L, M and N. **[5]**

END OF QUESTION PAPER